The Lorenz 96 model

1 Introduction

In 1996, Edward Norton Lorenz[1] of "Lorenz attractor fame"[2] described another chaotic system, now know as the Lorenz *96 model* which consists of $N \in \mathbb{N}, N \geq 4$ coupled differential equations of the form:[3]

$$\dot{x}_i = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F \tag{1}$$

with periodic "boundary conditions" (so to speak)

$$x_0 = x_N$$
$$x_{-1} = x_{N-1}$$
$$x_{N+1} = x_1$$

and a forcing constant F which is typically set to $F = 8$. All equations (1) can be scaled by a common scaling factor $\lambda = \frac{1}{20}$, yielding

$$\dot{x_0} = 20(x_1 - x_2)x_3 - x_0 + F^* \tag{2}$$
$$\dot{x_1} = 20(x_2 - x_3)x_0 - x_1 + F^* \tag{3}$$
$$\dot{x_2} = 20(x_3 - x_0)x_1 - x_2 + F^* \tag{4}$$
$$\dot{x_3} = 20(x_0 - x_1)x_2 - x_3 + F^* \tag{5}$$

for $N = 4$ with a forcing constant F^*.

2 Implementation

The implementation of equations (2), (3), (4), and (5) is straightforward and shown in figure 1.

[1] 23.05.1917–16.04.2008
[2] See application note #2 from 13.09.2016, `https://analogparadigm.com/downloads/alpaca_2.pdf`.
[3] See [Lorenz 1996, pp. 4 f.] and [Sterk et al. 2017].

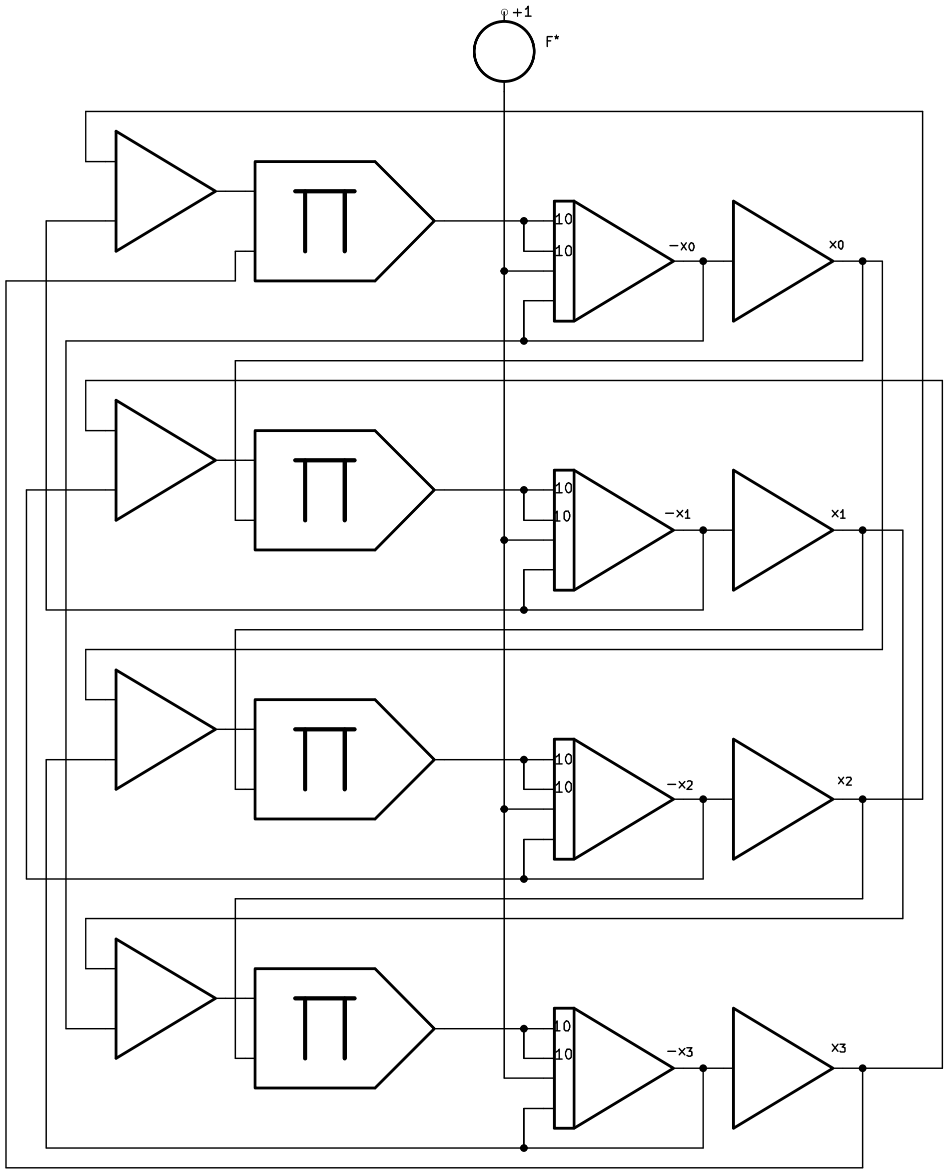

Figure 1: Implementation of the scaled LORENZ 96 system with $N = 4$

Analog values					
point attractor		cyclic attractor		chaotic attractor	
$F^* \geq$	$F^* \leq$	$F^* \geq$	$F^* \leq$	$F^* \geq$	$F^* \leq$
0	0.051	0.052	0.627	0.627	0.77

Table 1: Influence of F^*

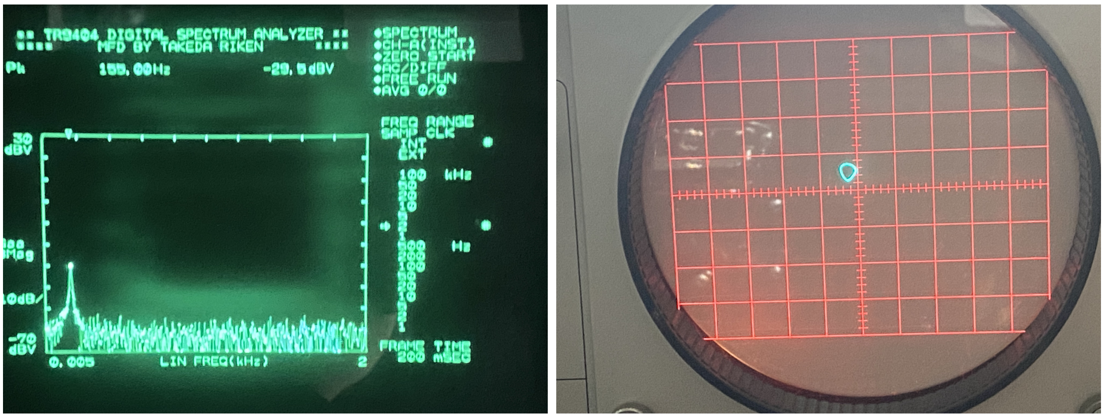

Figure 2: Start of the periodic attractor

3 Results

The system shows interesting behavior for varying F^*, with three attractor types shown in table 1.

Figures 2, 3, 4, and 5 show the behavior of the system for various values of F^*. On the right are phase space plots with an FFT analysis on the left. The switch from periodic to chaotic behavior can be clearly seen in the spectrum which starts to resemble more and more that of a noise signal.

The variation of the forcing constant F^* can be automated by integrating over a suitably small constant c such as $c = -0.2$ yielding a linear ramp $F^*(t)$ running from 0 to 1 in 5 seconds with the integrator set to $k_0 = 1$. Plotting $x(t)$ against $F^*(t)$ yields a simple bifurcation plot of the LORENZ 96 system as shown in figure 6.

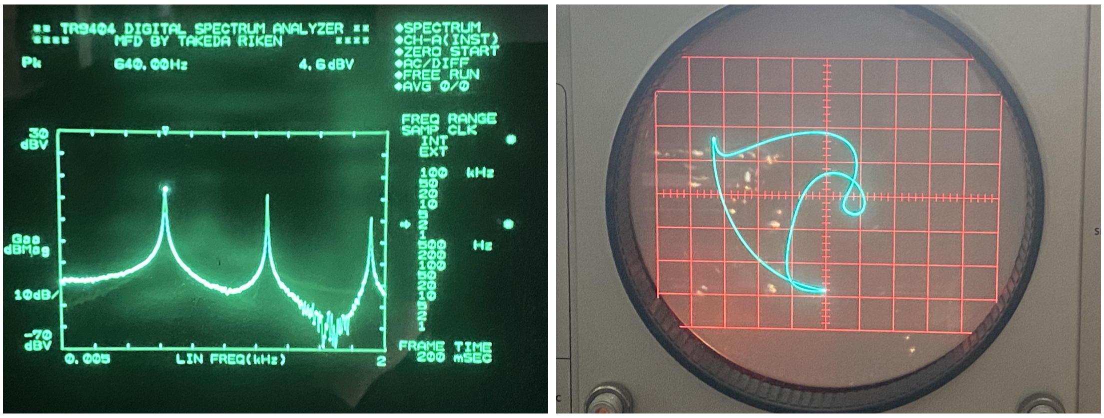

Figure 3: End of the periodic attractor

References

[LORENZ 1996] EDWARD NORTON LORENZ, "Predictability – a Problem Partly Solved", in *Proceedings on Seminar on Predictability*, 1, pp. 1–18

[STERK et al. 2017] A. E. STERK, D. L. VAN KEKEM, "Predictability of Extreme Waves in the Lorenz-96 Model Near Intermittency and Quasi-Periodicity", in *Complexity*, 2017, Article ID 9419024, https://doi.org/10.1155/2017/9419024

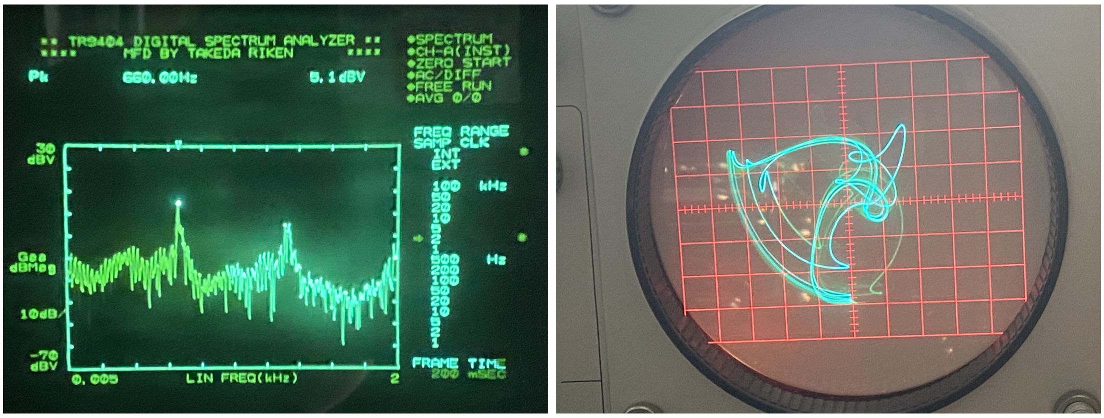

Figure 4: Start of the chaotic attractor

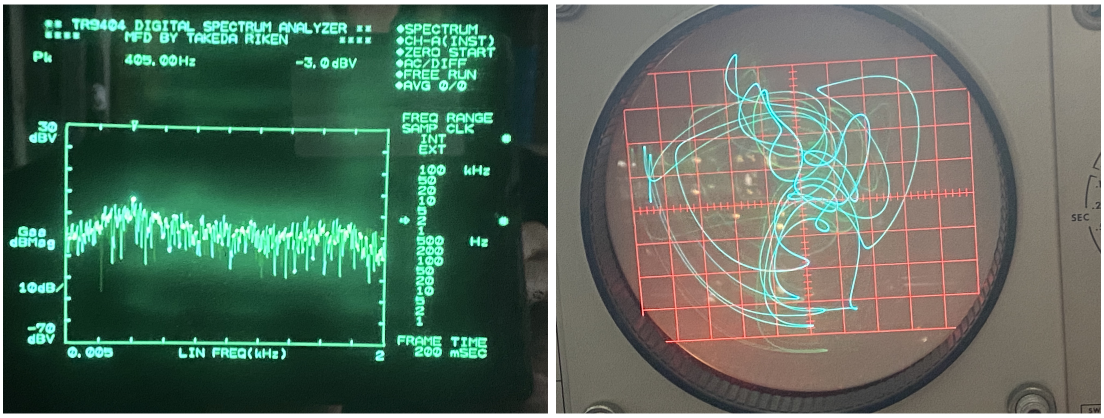

Figure 5: End of the chaotic attractor (maximum F^*)

Analog Computer Applications

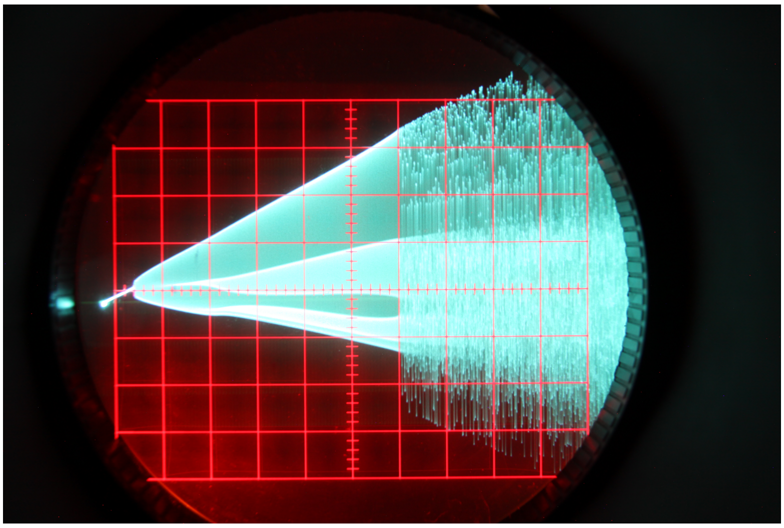

Figure 6: Simple bifurcation diagram of the Lorenz 96 system

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Issue #61, 05-JAN-2026