Analog Computer Applications
Application Note #12
Undergraduate Engineering Physics STEM Project

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Pendulum: A Tale of Two Terms
(A Simulation)

1   Introduction

This analog computer simulation will compute and display the angular position of a simple pendulum by replacing a sine function with the two-term, Taylor series expansion, $\sin(\theta) \cong \theta - \theta^3/3!$ The period of oscillation will be obtained from the oscilloscope display (see figure 3). It is assumed that the reader is familiar with the differential equation governing the angular displacement for this pendulum. Although air resistance will be ignored for this application, it will be included in the future. Stay tuned!

2   Mathematical modeling

Starting with the simple pendulum equation,

$$\ddot{\theta} = -g/L \sin(\theta),$$

replacing $\sin(\theta)$ with $\theta - \theta^3/3!$ instead of the 'usual' θ, setting $g/L = 1$ for simplicity, and omitting units for clarity,

$$\ddot{\theta} = -(\theta - \theta^3/3!)$$

Including the cubic expression, a wider swing for the pendulum is permitted as demonstrated below:

Letting $\theta = \pi/4$ (or 45º),

$\pi/4 - (\pi/4)^3/3! \cong 0.705$ and $\sin(\pi/4) = \sqrt{(2)}/2 \cong 0.707.$ Thus,

$0.705 \cong 0.707$, which is close enough! 😊

However, if just θ were used,

$\pi/4 \cong 0.785$ and $\sin(\pi/4) \cong 0.707$

$0.785 \neq 0.707$, which is not close enough! ☹

Returning to the differential equation and including initial conditions,

$$\ddot{\theta} = -(\theta - \theta^3/3!) = -\theta + \theta^3/3! \text{ with } \dot{\theta}(0) = 0 \text{ and } \theta(0) = \pi/4$$

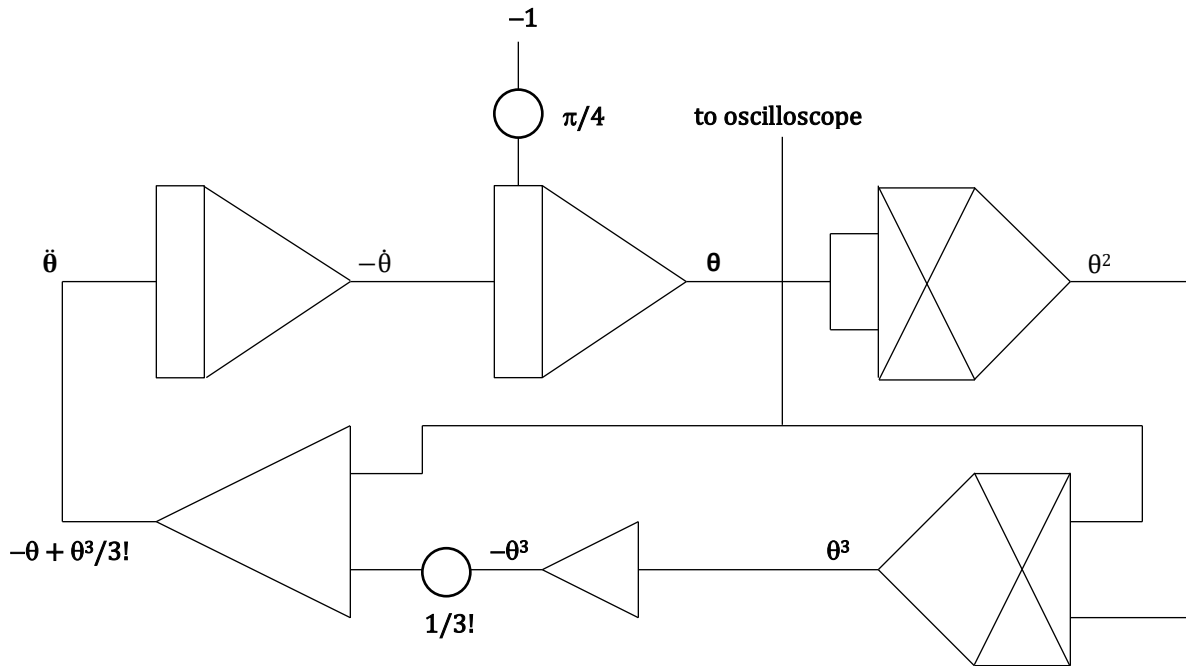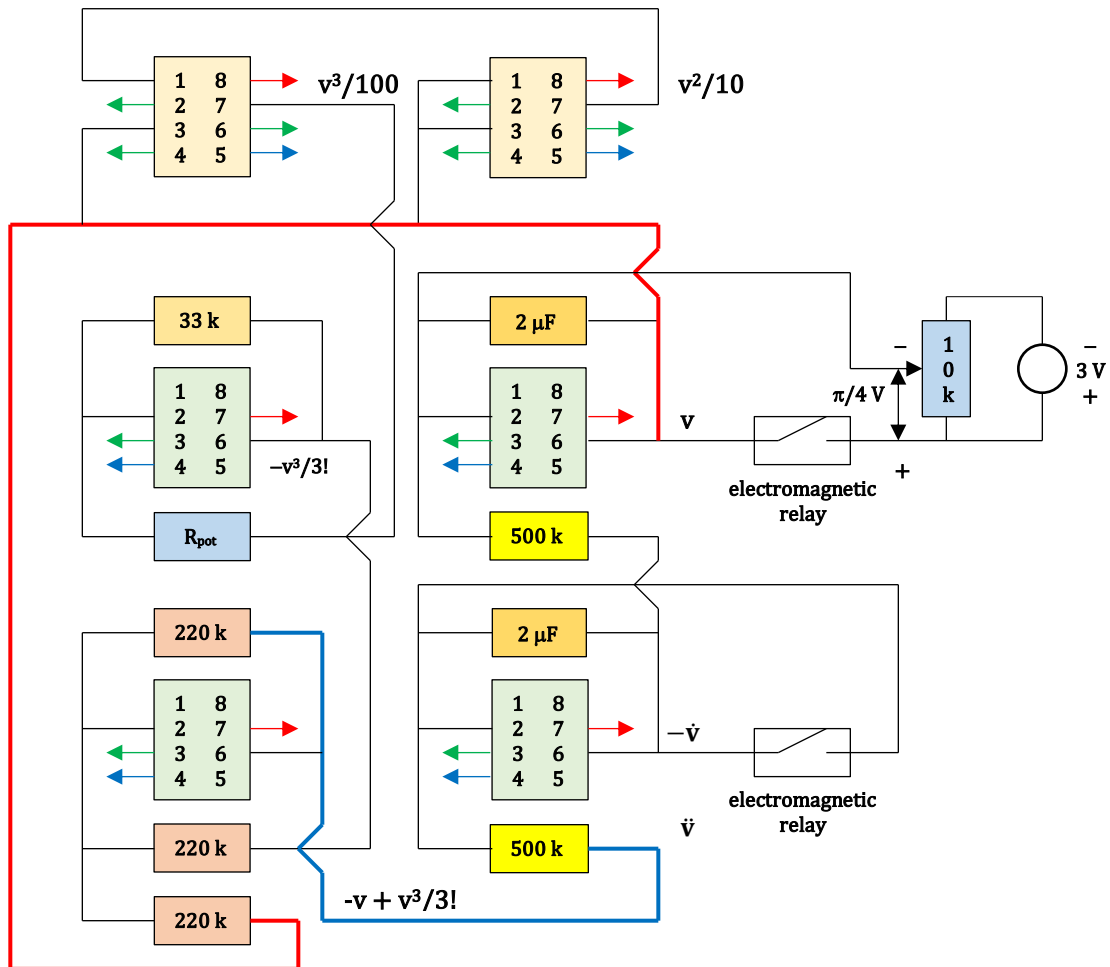## 3a   Computer setup (patch cord version

Figure 1: Computer setup for Pendulum

3b   Computer setup (IC and discrete component version)

Figure 2: Basic breadboard layout

Arrow Color Code:
Red arrows connect to V_{CC}, not shown in diagram.
Blue arrows connect to V_{EE}, not shown in diagram.
Green arrows connect to ground.

Comments:
θ has been replaced by v (voltage).
R_{pot} adjusted to obtain 1/3!
500 k = two, 1 MΩ resistors in parallel.
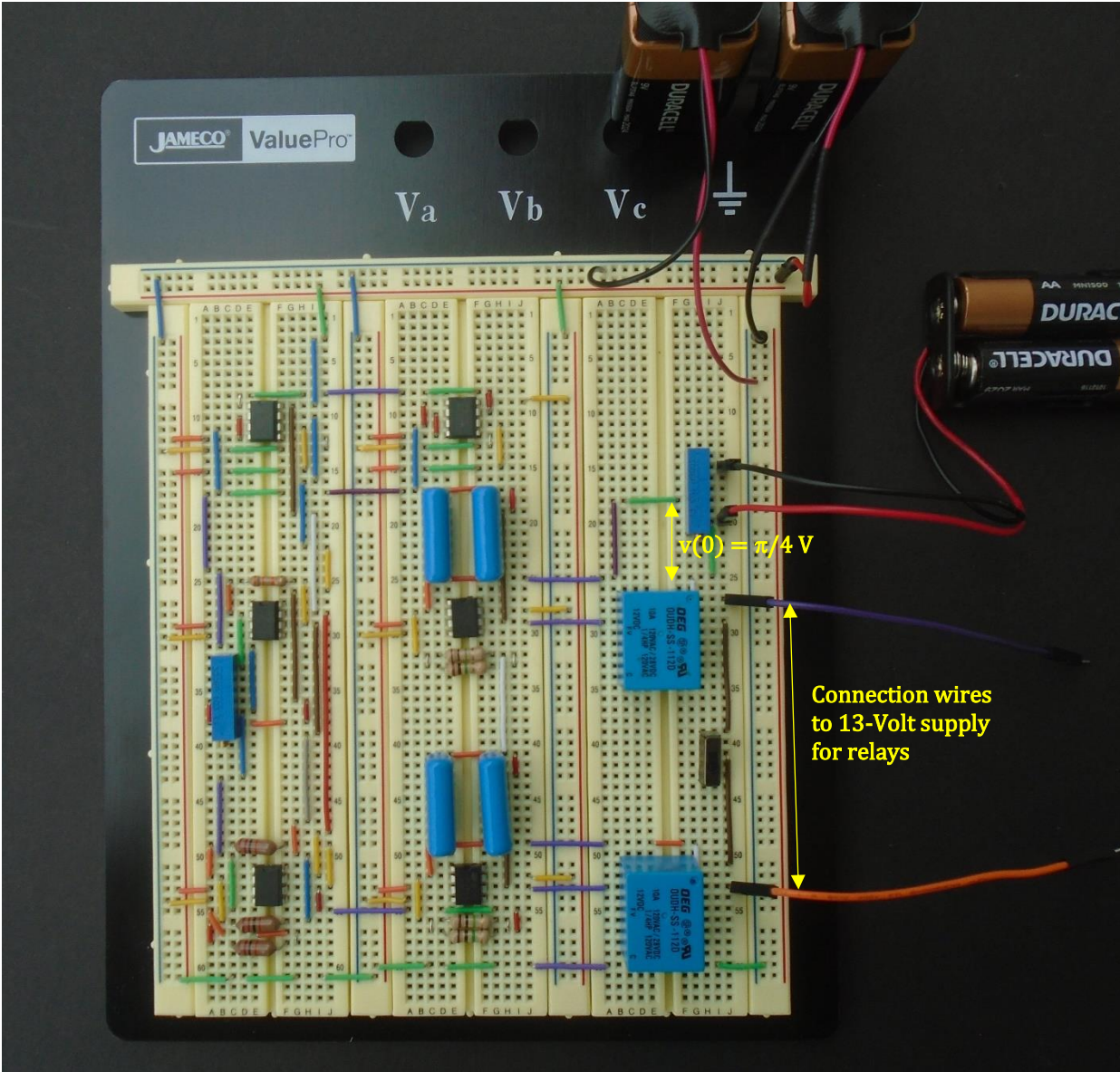2 μF = two, 1 μF in parallel.

Figure 3: Analog computer circuit
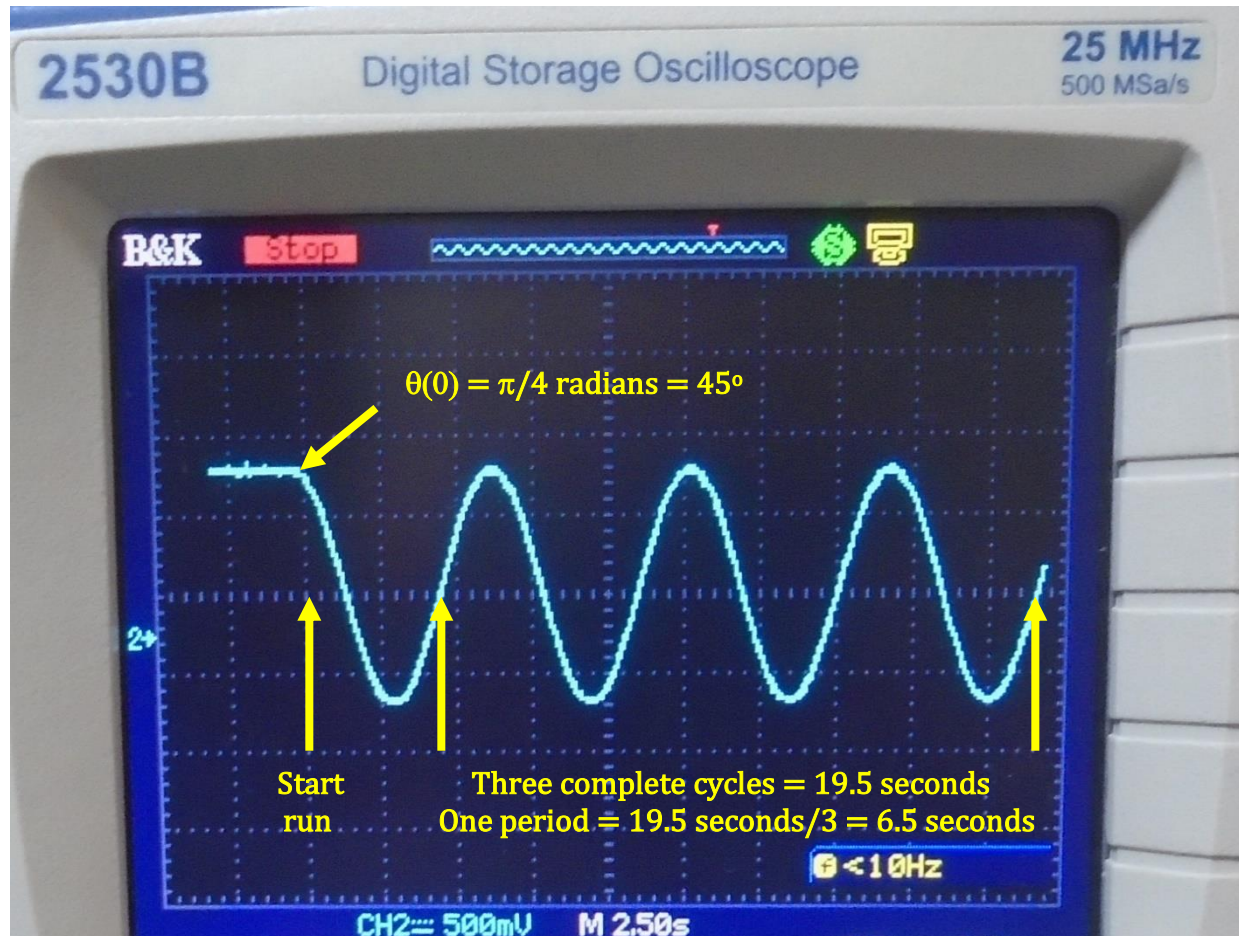
4   Results

Figure 4: Angular pendulum swings*

*For this application note, the oscilloscope display was produced during a single run using a differential equation analog computer constructed from operational amplifiers, analog multipliers, and discrete components with tolerances up to 20%.