Zombie Apocalypse Now[1]

1 Introduction

How could one not like zombie movies? It was about time that someone, Robert Smith? and his collaborators, shed some light on zombie attacks from a mathematical point of view (see [Smith?(2014)]). Phil Munz (see [Munz(2014)]) applied a well-known coupled set of differential equations to this particular problem, that was independently developed by Alfred James Lotka and Vito Volterra in the late 19th/early 20th century. Although Volterra and Lotka were interested in (closed) eco-systems, their equations are ideally suited to model a world infested by zombies. Whenever there is a problem which is readily described by (coupled) differential equations, an analog computer is the ideal tool to tackle them as we will do in the following.

2 Programming

The mathematicsl model itself is quite straightforward and consists of the following two coupled differential equations:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha h - \beta hz \tag{1}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \delta hz - \gamma hz - \zeta z \tag{2}$$

[1] Version 1.1

h and z represent the number of humans and zombies, respectively. h_0 and z_0 are the initial conditions. The remaining parameters are

α: Growth rate of the human population (birthrate).

β: Factor describing the rate at which humans are killed by zombies.

δ: Growth factor of zombie population due to zombies killing humans and thus transforming them into zombies.

γ: Rate at which zombies are killed by humans.

ζ: Normal "death" rate of the zombie population.

Figure 1 shows the resulting program for the two equations (1) and (2) while figure 2 shows the program as implemented on an Analog Paradigm Model-1 analog computer.

3 Results

Figure 3 shows a typical result obtained with the setup described above. The computer was run in repetitive mode with the time-constants of the integrators set to $k_0 = 10^3$. Additionally, only integrator inputs with a weight of 10 were used, further speeding up the simulation by another factor of 10. The IC-time was set to short, and the OP-time to 60 ms. The oscilloscope was explicitly triggered with one of the TRIG-outputs of the CU to obtain a stable, flicker-free display.

The parameters were derived experimentally by playing with the various coefficients until a stable behaviour was obtained. The output shown

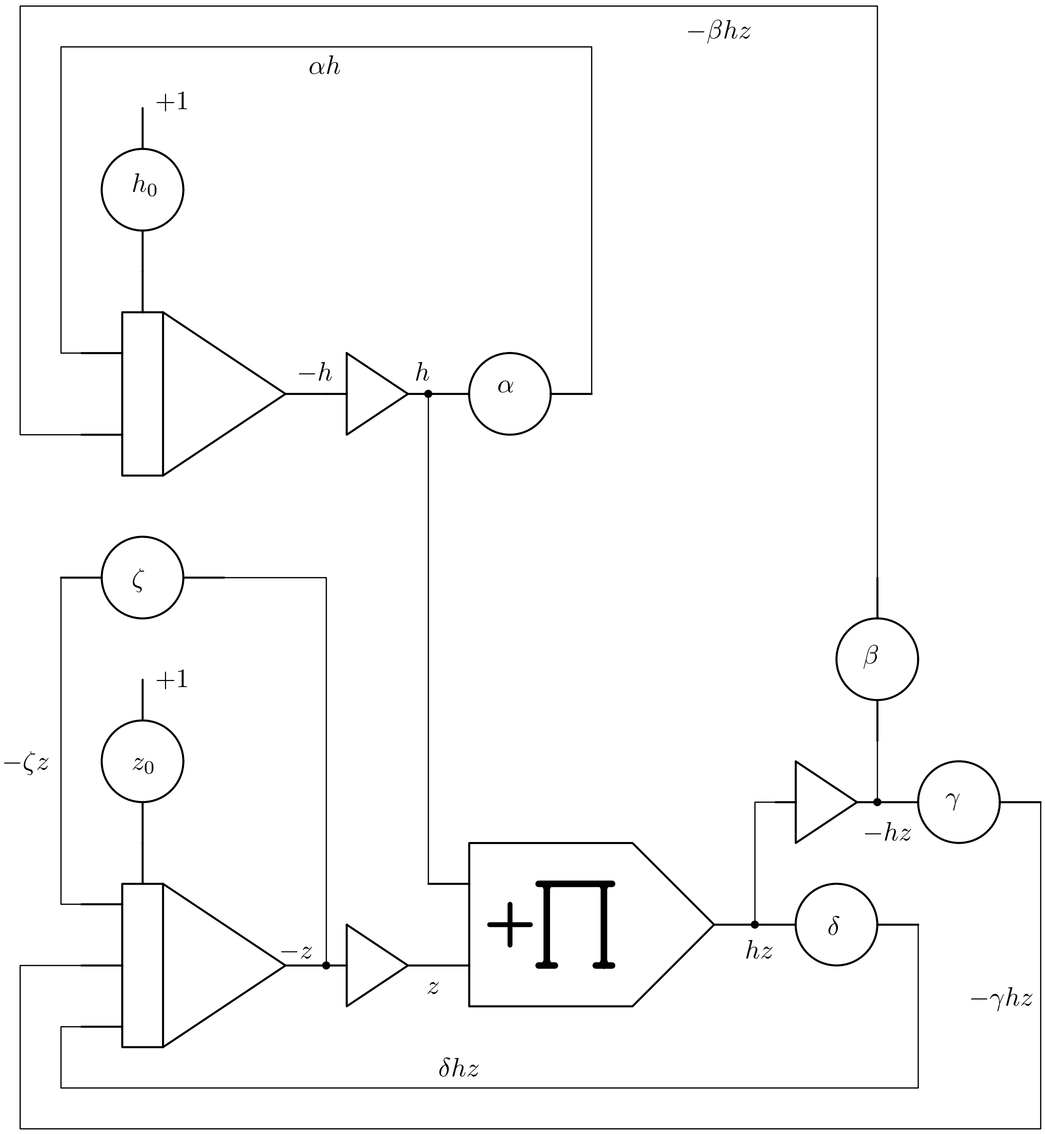

Figure 1: Analog computer program for equations (1) and (2)

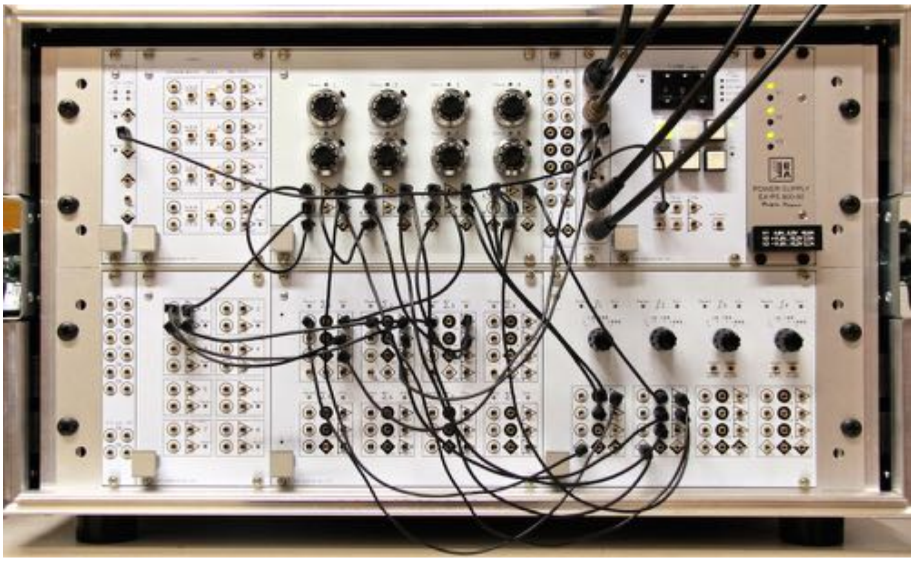

Figure 2: Zombie simulation on an Analog Paradigm Model-1 analog computer

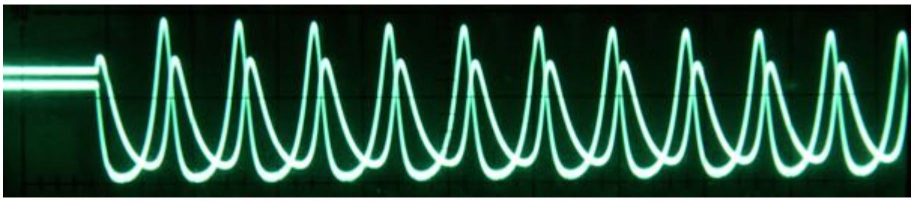

Figure 3: Results of a typical zombie simulation

was generated with $h_0 = z_0 = 0.6$, $\alpha = 0.365$, $\beta = 0.95$, $\delta = 0.84$ (very successful zombies, indeed), $\gamma = 0.44$, and $\zeta = 0.09$.

References

[Smith?(2014)] Robert Smith?, *Mathematical Modelling of Zombies*, University of Ottawa Press, 2014

[Munz(2014)] Phil Munz, "When Zombies Attack! Alternate Ending", in [Smith?(2014)], pp. 45–55